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SELF-SIMILAR HEATING REGIME UPON DESTRUCTION OF THE SURFACE OF MATERIALS

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The applicability of the dependence $\Delta^* \approx K\sqrt{a\tau}$ in mass transfer from the surface of heat insulating materials is experimentally demonstrated. A formula for calculating the temperature coefficient K is suggested.

In the classical theory of heat conduction the notion of self-similar heating is widely used; this means that a dimensionless Fourier number becomes the single variable determining the process of heat propagation. It is believed that to establish this regime, it is necessary that the temperature of the outer, heated surface be maintained constant and that mass transfer from the surface either by nonexistent, or that its rate be inversely proportional to the square root of the time.

However, the self-similar solution for a semiinfinite body not subject to destruction and with constant temperature $T_w = T_p$ = const [1]

$$\Theta^* = \frac{T^* - T_0}{T_w - T_0} = \operatorname{erfc}\left(\frac{y}{2\sqrt{a\tau}}\right) \tag{1}$$

satisfies even more complex variants of thermal loading. For instance, according to the calculations by A. V. Vasin, when the surface temperature changes trapezoidally, the depth of heating δ_T is described by the "almost" self-similar expression

$$\delta_T \approx K\sqrt{a\tau}, \tag{2}$$

if $m/(e + n) \geq 2$. Here,

$$K = \Theta^{*-0.3}; \tag{3}$$

e, n are the heating and cooling sections, respectively, and m is the section with the temperature T_w = const.

In distinction to the classical self-similar regime, in the experiments of [2] a quasisteady velocity of surface mass transfer was observed. The time of establishing such a velocity was about one fifth of the time τ_δ, nevertheless, in the time interval $\tau \leq \tau_\delta$ the distance through which the isotherm of phase transformations passed obeyed the dependence type (2). In these experiments the quasisteady velocity of mass transfer changed to a multiple while the surface temperatures were practically equal. However, the overall amount of heated and removed material within the same time of heating remained the same within the accuracy of the experiment. Such conditions of destruction were attained by testing specimens in air and nitrogen plasma and under radiative heating. It may consequently be assumed that the regularity of change in velocity of the outer surface has no effect on the rate of displacement of the isotherm of phase transformations in the time interval $\tau \leq \tau_\delta$.

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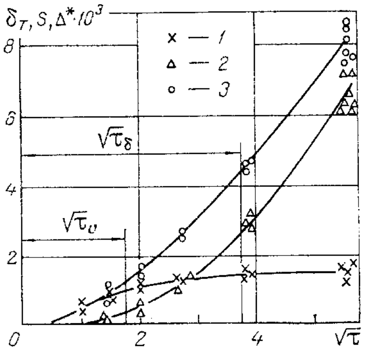

Fig. 1

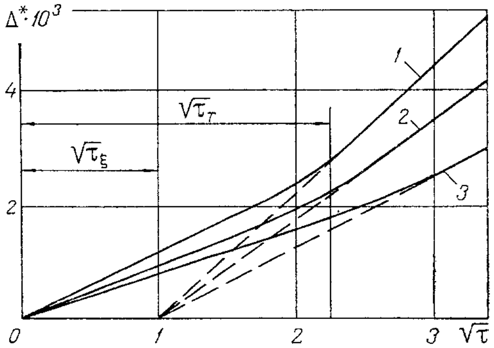

Fig. 2

Fig. 1. Dependence of the parameters of destruction of an alloyed quartz glass ceramic on the heating time: 1) depth of heating; 2) linear mass transfer; 3) sum of linear mass transfer and depth of heating. δ_T, S, Δ^*, m; $\sqrt{\tau}$; $\sec^{1/2}$.

Fig. 2. Dependence of the distance covered by the isotherms on the heating time for specimens of asbestos textolite: 1-3) $\Theta^* = 0.03$, 0.07, and 0.23, respectively (the dashed lines determine the nominal point of intersection of the dependences 1-3 with the axis of abscissas).

In other words, in the entire time interval $\tau \leq \tau_\delta$ (before a constant depth of heating is established), mass transfer $S(\tau)$ and the thickness of the heated layer $\delta_T(\tau)$ (Fig. 1), if counted off the true surface of the body, may change in time according to completely different regularities. However, in sum, they will yield a value that is satisfactorily described by the "self-similar" solution.

If we take into account the time within which the "self-similar" regime of heating τ_ξ becomes established, we may write

$$\Delta^* = K\sqrt{a}\,(\sqrt{\tau} - \sqrt{\tau_\xi}) \tag{4}$$

for $\tau_T \leq \tau \leq \tau_\delta$.

Since in [2] the depth of heating was measured according to the isotherms of phase transformation, it is of interest to verify the applicability of dependence (4) in a broader temperature range.

For this purpose we carried out experiments measuring the temperature field with the aid of thermocouples in a specimen of asbestos textolite tested in a gas generator jet with a heat flux of 3500 kW/m^2. It can be seen from Fig. 2 that the path covered by each isotherm from the instant τ_T onward satisfies expression (4). The graph in Fig. 1 (curve 3), which also confirms the linearity of the dependent $\Delta^* = f(\sqrt{\tau})$, was obtained from the results of tests of specimens of alloyed quartz glass ceramics in a subsonic jet of an electric-arc heater [3] when he heating tme was changed from 1 to 30 sec. The depth of heating was checked according to the isotherm corresponding to the melting point of quartz glass ceramics ($T^* \approx 2000$°K, $\Theta^* = 0.7$). At this temperature the porosity of the specimens changes, and the additives — chromium oxides — become partly dissolved, as a result of which the material abruptly changes its color. The depth of heating was measured with a microscope MBS-9. Processing of the results showed that the experimental error did not exceed 10%.

Since it follows from (2) that $K \approx \delta_T/\sqrt{a\tau}$, the question arises to what extent K depends on the mentioned parameters. From the solution of the problem of heating a semiinfinite insulated body without mass transfer from the surface in [4, 5] we obtained expressions type (1) for calculating K with different regularities of the change of T_w. The calculations, carried out in the range of change of the thermal diffusivity by two orders of magnitude, showed that the coefficient K depends solely on Θ^* and the regularity of the change of T_w on the surface, and that K is the larger, the more slowly T_w changes.

When the surface is destroyed under the effect of a constant thermal flux, after the time $\tau = \tau_T$ a constant temperature T_p is established on the surface, and from this instant

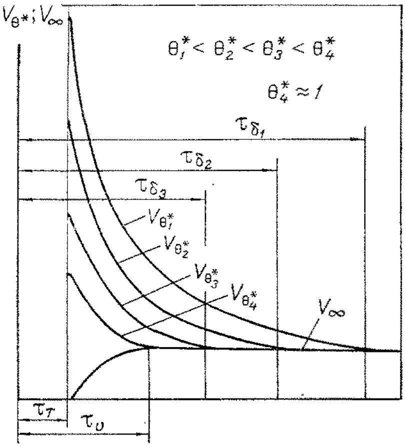

Fig. 3

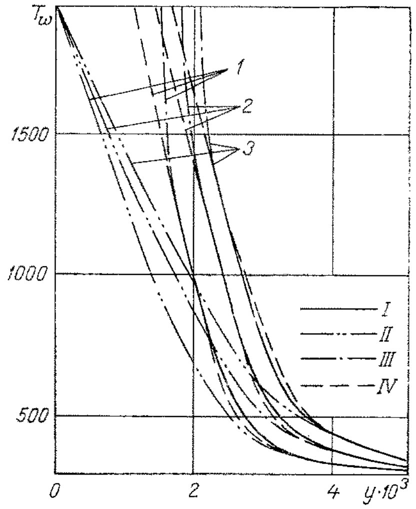

Fig. 4

Fig. 3. Diagram of the change of the velocities of the isotherms V_{Θ^*} and of the surface V_∞ in dependence on the heating time.

Fig. 4. Experimental and theoretical temperature profiles in a specimen of asbestos textolite: 1–3) heating time 7, 9, 11 sec, respectively; I) experiment; II) calculation of (1); III) calculation by (3); IV) calculation by (7). T_w, °K; y, m.

onward Δ^* is already a linear function of $\sqrt{\tau}$. When we differentiate (2), we obtain for the velocity of the isotherm Θ^* the expression

$$V_{\Theta^*} = \frac{K\sqrt{a}}{2\sqrt{\tau}}. \tag{5}$$

Since it was experimentally shown (see Fig. 1) that (2) applies to isotherms with Θ^* up to 0.7, it may be assumed that (2), (5) are also applicable to surface isotherms with $\Theta^* \approx 1$. However, the time of establishing the depth of heating τ_δ depends on which isotherm is chosen as basis [4]. In proportion to the increase of temperature ($\Theta^* \to 1$) the value of $\tau_\delta \to \tau_V$, i.e., the time of establishing a constant velocity of mass transfer (since τ_V also characterizes the surface isotherms attaining a quasisteady velocity). The smaller Θ^* is, the larger is τ_δ. Thus, according to [4], with m = 0.1, $\tau_\delta/\tau_T \sim 400$, and with $\Theta^* = 1$ and the same m = 0.1, $\tau_\delta/\tau_T \sim 40$.

With increasing m the ratio τ_δ/τ_T decreases to 2-5. The range of the existence of the dependence $\Delta^* \approx K\sqrt{a\tau}$ may therefore be very small (especially for surface isotherms).

According to (5) the velocity of any isotherm has to decrease in proportion to $1/\sqrt{\tau}$, and its maximum value corresponds to the instant τ_T. At that same instant linear mass transfer begins, and its velocity gradually increasing, attains its quasisteady value $\bar{V}_\infty$ at the instant τ_V (Fig. 3). If we apply the same schema to the surface isotherms, too, it is indispensable that in distinction to the classical self-similar solution, the coefficient K not be equal to 0, even when $\Theta^* = 1$. In addition, in the time interval $\tau_T \leq \tau \leq \tau_\delta$ the velocity of the surface isotherms is greater than V_∞ only if its value is sufficiently high.

Since with $\Theta^* = 1$, K = 0 in accordance with (1), (2), the question arises first of all whether (1) may be used for calculating the coefficient K under conditions of mass transfer from the surface. When we determine the depth of heating according to the isotherm $\Theta^* = 0.1$, then formula (1) yields a value equal to 2.5. In [4] an expression was obtained for calculating the depth of heating; it had the form

$$\delta_T = \frac{a}{\bar{V}_\infty} \ln \frac{T_w - T_0}{T^* - T_0}. \tag{6}$$

The numerical calculations carried out in this work show that, e.g., with $\Theta^* = 0.1$, $4a/\overline{V}_\infty^2$, and $\Delta^*(\tau_\delta) = S(\tau_\delta) + \delta_T(\Theta^*) = 5.1a/\overline{V}_\infty \approx 2.5\sqrt{a\tau_\delta}$, i.e., with $\Theta^* = 0.1$, $K = 2.5$, which is in agreement with the calculation of (1).

To find the dependence of K on Θ^* in the entire range $0 < \Theta^* < 1$, we use the temperature field obtained earlier in a specimen of asbestos textolite with the aid of thermocouple measurements.

Figure 4 presents the experimental and theoretical temperature profiles for the instants 7, 9, and 11 sec. The calculation was carried out by (4), the data on thermal conductivity and heat capacity of asbestos textolite that were used taken from [6]. The time τ_ξ was found from the graphs in Fig. 2. In addition to (1), (3) in the calculation of K, we also used the linear approximation of the form

$$K = -\frac{1}{K_{T_\mathrm{p}}}\Theta^* + \frac{K_{T_\mathrm{p}}^2}{1 - K_{T_\mathrm{p}}}, \tag{7}$$

where $K_{T_\mathrm{p}} \approx 0.74$ and equal to K for $\Theta^* = 1$.

The obtained results show that with $\Theta^* < 0.2$-0.3, dependence (1) yields good agreement between calculation and experiment. When $\Theta^* > 0.2$, it is better to use expression (7).

It is interesting to note that in expression (7) a single value of K_{T_p} is possible because when $\Theta^* = 1$, it becomes a cubic equation

$$2K_{T_\mathrm{p}}^3 - K_{T_\mathrm{p}}^2 + K_{T_\mathrm{p}} - 1 = 0,$$

and solving it, we find the numerical value $K_{T_\mathrm{p}} = 0.73898367 \approx 0.74$.

Thus, in the general case we may approximately take it that $\Delta^*(\tau) = S(\tau) + \delta_T(\tau) \approx K\sqrt{a\tau}$, where, in distinction to the classical self-similar solution, $K \neq 0$ even when $\Theta^* = 1$ because the isotherm shifts in relation to the initial dimension of the body. It is possible that the factor K depends on the mechanism of destruction. Thus a change of $m = c_\mathrm{p}(T_\mathrm{p} - T_0)/\Delta Q$ is bound to lead to a nonmonotonic change of K. However, in the range $0.1 \le m \le 1$ (which is of particular practical interest) this influence does not exceed ±15%.

NOTATION

T_w, temperature of the surface of the material; T^*, temperature of the examined isothermal surface; T_p, temperature of destruction of the surface; T_0, temperature of the nonheated material; τ, heating time; τ_T, τ_V, τ_δ, times of establishing the quasisteady values of the temperature of the surface, the velocity of mass transfer, and depth of heating, respectively; δ_T, depth of heating; $S(\tau)$, linear mass transfer of material; Δ^*, sum of the thicknesses of the heated material and of the material removed from the surface; K, temperature coefficient; $K_{T\mathrm{p}}$, temperature coefficient for $\Theta^* \approx 1$; a, thermal diffusivity; V_∞, V_{Θ^*}, velocity of displacement of the outer surface and of the isotherms; m, parameter of the thermal effectiveness of material in the process of destruction; c_p, heat capacity; ΔQ, thermal effect of destruction.

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